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ASSESSING DAMPING UNCERTAINTY IN SPACE STRUCTURES WITH FUZZY SETS

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ABSTRACT

NASA has been interested in the development of methods for evaluating the predictive accuracy of structural dynamic models. This interest stems from the use of mathematical models in evaluating the structural integrity of all spacecraft prior to flight. Space structures are often too large and too weak to be tested fully assembled in a ground test laboratory. The predictive accuracy of a model depends on the nature and extent of its experimental verification. The further the test conditions depart from in-service conditions, the less accurate the model will be. Structural damping is known to be one source of uncertainty in models. This paper explores the uncertainty in damping to evaluate the accuracy of dynamic models. A simple mass-spring-dashpot system is used to illustrate a comparison among three methods for propagating uncertainty in structural dynamics models: the First Order Method, the Numerical Simulation Method and the Fuzzy Set Method. The fuzzy set method is shown to bound the range of possible responses and thus to provide a valuable limiting check on the First Order Method near resonant conditions. Fuzzy methods are a relatively inexpensive alternative to numerical simulation and they can be used to classify uncertain parameters into useful groupings.

INTRODUCTION

With the availability of high-speed digital computers and finite element modeling, it has become possible to model highly complex structural systems, such as the Space Shuttle, in great detail with tens of thousands of structural degrees of freedom (DOF). Structural dynamic models are greatly reduced, however, depending on their application. For example, dynamic analysis models may be reduced to thousands of DOF, test support models to hundreds of DOF, and control system models to tens of DOF. One of the chief concerns in model reduction is loss of accuracy, particularly in the very low-order models which represent the structural "plant" in controls applications. There are numerous other sources of inaccuracy as well, which can only be evaluated by testing the structure.

Testing for purposes of dynamic model verification usually involves a modal survey. While it used to take weeks or even months to conduct a modal survey using tuned sine dwell and analog data processing, the same number of modes can now be obtained in a matter of days using random vibration and digital data processing. Again, the digital computer has played a vital role in the development of this technology. Unlike the large mainframe computers used for analyzing finite element models, it is the minicomputers and microprocessors which over the past ten years have given impetus to the growth of experimental modal analysis. Experimentally derived modes are routinely compared with analytically predicted modes as a means of verifying an analytical model.

For the most part, analytical model verification is still performed intuitively, by trial and error. Experimental mode shapes and frequencies are compared with analytical predictions and the model is adjusted by hand in an effort to bring it into agreement with experimental data. Very often, the experimental modes are used directly in subsequent analysis rather than attempting the time-consuming (and sometimes unsuccessful) task of adjusting a model to match the data. There are many cases where the experimental modes cannot be used directly, however, as in the case of large space structures which are too large and/or too weak to be ground tested in their entirety. Models must then be relied upon to predict dynamic behavior, and the accuracy of these predictions is of major concern.

One of the problems confronting engineers today is, that while the tools for structural design, analysis and testing have individually matured over the past two decades, it is still not possible to predict with confidence how well a structure will perform in a given environment without direct experimental verification, i.e. without actually testing the operational configuration of the structure in an adequate simulation of that environment. In the case of large space structures, neither of these conditions can be met. The thermal, atmospheric and gravitational environment of space cannot as yet be adequately simulated in a ground test laboratory. Secondly, the fully assembled structures are too large to be tested in an earth-gravity environment except by either substructure or subscale testing. In many cases their configurations will change as transport vehicles dock and separate, as appendages are repositioned, as supplies are consumed, and as the structures grow or are otherwise altered to accomplish various space missions. Knowledge about the accuracy of a model is important information for the design of a control system; some inaccuracy can be tolerated by a robust controller, but there are tradeoffs between robustness and performance. The greater the uncertainty in the structural model, the poorer the control of the structure.

Two needs are therefore recognized: (1) to develop a means of evaluating the predictive accuracy of structural dynamic models when the structure cannot be tested under in-service conditions, and (2) to develop methods for enhancing the predictive accuracy of a model through a suitable program of ground testing. On-orbit testing will ultimately be required to obtain an accurate model of the "as-built" structure in space. However, the task of on-orbit identification will be greatly facilitated (perhaps only possible) by having first verified major portions of the structural model through ground testing.

This paper illustrates some of these ideas by focusing on the uncertainty of damping in structural dynamics models. The estimated modal damping matrix corresponds to the test modes. In general, a different modal damping matrix is obtained when the estimated damping matrix is transformed to the coordinate space of the analytical modes. The difference between the two provides one measure of damping uncertainty. The paper also demonstrates by numerical example three methods for propagating the uncertainties in modal mass, stiffness and damping forward through the model to evaluate the accuracy of response predictions, and backward to evaluate the uncertainties of model parameters. These three alternative methods for propagating uncertainty are the: First Order Method, Numerical Simulation Method and Fuzzy Set Method.

UNCERTAINTY IN DAMPING

The normal mode method is widely used for dynamic analysis of linear structures. By enabling the equations of motion to be written in terms of modal coordinates, solutions are more readily determined. Fortunately, structural damping tends to be small so that the classical undamped modes have a useful physical interpretation. It is common practice to introduce damping only after the equations have been transformed to modal coordinates. In this case, viscous damping is typically assumed and the modal damping matrix is taken to be diagonal. The elements along the diagonal are related to the percent of critical damping for each mode, while the rest of the matrix is neglected assuming that the modes are not coupled by damping forces in the structure. This assumption is valid whenever the modal frequencies are not closely spaced [1].

Although justification may be found for neglecting these terms in some analyses, there are times when this assumption is inappropriate. For example, when modal synthesis is employed to combine substructure characteristics in deriving the equations of motion for a complete structure, and linear viscous damping is taken to represent the dissipative mechanism of the structure, the full modal damping matrices are required for each substructure. Since the off-diagonal terms are likely to be of the same order as the diagonal ones, they too will influence the modal damping being computed for the complete structure.

The full modal damping matrix will also be useful in adjusting experimentally determined modal damping for structural models which must be revised to account for differences between earth and space environments. Such differences may include

- mass, stiffness and damping of suspension systems
- gravity loading
- thermal loading
- air damping

There are now several methods available for estimating the full modal damping matrix and for extracting complex modes from measured structural response [2-5].

METHODS FOR PROPAGATING UNCERTAINTY

There are a number of ways in which uncertainty can be propagated through a model. Theoretically, if probability distributions were known for the parameters of a model, and a functional relationship existed between the parameters and some desired response characteristic such as frequency response, then it would be possible to determine the probability distribution of that response characteristic. From a practical standpoint, however, this approach is not feasible. Probability distributions for the model parameters are rarely if ever available, and even if they were, the task of combining them to obtain the probability distributions of response would be exceedingly difficult. Fortunately, more practical alternatives do exist. Three are discussed in the following subsections, and then compared for a simple numerical example.

First Order Statistical Method

The First Order Statistical Method is perhaps the simplest, least expensive and most familiar approach. First order methods are based on linearization and are best suited to problems involving either linear or weakly nonlinear relationships among the parameters and input-output variables of the problem.

First order statistical methods are based on the principle of linear covariance propagation, or the linear transformation of covariance matrices from one set of variables to another. For example, suppose that r denotes a vector of random variables. These random variables might represent selected mass and stiffness parameters of a structural model which are not precisely known. The expected values of these random variables may be designated by the vector μ_r. The covariance matrix of the vector r is then

$$E[(r-\mu_r)(r-\mu_r)^T] = S_{rr} \tag{1}$$

Suppose further that the vector u represents a second set of random variables (e.g. eigenvalues and eigenvectors) related to r by $u = f(r)$. The random variables, u, can be expressed in terms of the random variables, r, using a Taylor series expansion about the mean of u, denoted μ_u.

$$u = \mu_u + \partial u/\partial r\,(r - \mu_r) + \ldots\ldots \tag{2}$$

If the higher order terms are neglected, the covariance matrix of u is given by

$$E[(u-\mu_u)(u-\mu_u)^T] = E\{\partial u/\partial r\,(r - \mu_r)\,(r - \mu_r)^T\,\partial u/\partial r^T\} = S_{uu} \tag{3}$$

or

$$S_{uu} = T_{ur}\,E[(r-\mu_r)(r-\mu_r)^T]\,T_{ur}^T = T_{ur}\,S_{rr}\,T_{ur}^T \tag{4}$$

where E denotes the expectation operator, and

$$T_{ur} = \partial u/\partial r \tag{5}$$

In particular, the jkth element of the rectangular partial derivative matrix, T_{ur}, is the scalar quantity

$$(T_{ur})_{jk} = \partial u_j/\partial r_k \tag{6}$$

It is desirable to make the inverse transformation from u to r as well as the direct transformation from r to u. However, whereas one can express u as an explicit function of r, the converse is not true; one cannot write the functional relationship $r=f^{-1}(u)$ in explicit form. Instead, r and S_{rr} are obtained by statistical estimation [6-8]. The inverse transformation of the covariance matrix S_{uu} to S_{rr} is given by

$$S_{rr} = [(T_{ur})^T\,S_{uu}^{-1}\,T_{ur}]^{-1} \tag{7}$$

In Equation (7) the dimension of u must be greater than or equal to that of r.

Equation (7) is useful in the evaluation of predictive accuracy. A method is available to derive S_{uu} from sets of predicted and measured modal data whenever u represents modal frequencies and displacements. From this information it is possible to obtain by direct transformation (Equation 4) the covariance matrices of frequency response, impulse response or other measures of performance which are dependent on these. It is also possible to obtain the corresponding covariance matrix of parameter estimates by the inverse transformation (Equation 7).

Numerical Simulation

Numerical simulation is conceptually the simplest method for propagating random uncertainty through a model. The model may be linear or nonlinear, and the random parameters of the model may be assigned any desired distribution. Unlike linear covariance propagation where only the first two central moments of the parameter distributions are propagated, the entire distributions are propagated in numerical simulation. The chief disadvantage is the computational effort required.

In numerical simulation (or Monte Carlo simulation), parameter values are selected at random, and the model is exercised to compute the response quantities of interest. The desired parameter distributions are obtained by first using a random number generator to generate a sequence of numbers uniformly distributed between zero and one. The resulting sequence of numbers is used in the simulation. Because of the usual large number of calculations required for accuracy, this type of numerical simulation is not practical for large structural dynamics models. It is useful, however, for treating isolated nonlinearities, and for applications involving simple models.

Fuzzy Set Method

Fuzzy sets offer an alternative to random variables for representing uncertainty. Numerous works explaining fuzzy sets are available in the literature [9]. Whereas the uncertainty of a random variable is measured in terms of probability, the uncertainty of a fuzzy set can be measured in terms of possibility. Probability implies random uncertainty; however, possibility can be used to measure non-random uncertainty. The degree of uncertainty in a fuzzy set is defined by its membership value. The membership of a fuzzy set measures the level of possibility and ranges from zero to one. The degree of membership in a set can be thought of as a measure of the "belongingness" of a particular variable to that set. Fuzzy sets are used quite often to describe "linguistic" variables (such as light, moderate, heavy, etc.) where the variable can have a vague, or fuzzy meaning. Unlike probability density functions which define the relative frequency of occurrence of a random variable as a function of the values which the random variable may assume, the membership function defines the range of possibility of a fuzzy number as a function of membership. In the case of a triangular membership function where the vertex has a membership of unity, the value of the fuzzy number corresponding to the vertex is interpreted as the deterministic value.

It is important to keep in mind that the concepts of a density function and a membership function are different. A density function is based on probability theory which in turn is postulated from "crisp set" mathematics. A crisp set merely defines the sample space of a random variable; the variable is either in the sample space (membership = 1) or it is not (membership = 0). A fuzzy set differs from a crisp set (sample space) by allowing for vagueness in the prescription of the boundaries of the sample space. It is also noted that crisp sets are special subsets of fuzzy sets, and that probability theory is a special subset of possibility theory.

With this distinction in mind, one can attempt to relate the membership function of a fuzzy set to the probability density function of a random variable. This will be shown to be a useful relationship in the sense that it provides a means of bounding the uncertainty of response predictions, particularly when structural response is a highly nonlinear function of the uncertain model parameters. In this situation, first-order methods tend to be unreliable, and simulation methods too costly.

The propagation of uncertainties using fuzzy sets involves computations with interval variables and functions [9]. For example, a variable, x, could have as its value a or b or 3.5, etc. which are real numbers. Similarly, an interval variable, denoted by X, will have as its value [a,b]. All arithmetic operations on interval numbers can be applied to interval variables. A function of the interval variable X = [a,b] can be defined by

$$Y = f(x) = \{f(x) \mid x \in X\} = \{f(x) \mid x \in [a,b]\} \qquad (8)$$

whose value usually would be an interval number. When f(x) is continuous and monotonic on X = [a,b], Y can simply be obtained by

$$Y = \{\min [f(a), f(b)], \max [f(a), f(b)]\} \qquad (9)$$

The Vertex Method can be used to propagate uncertainties whenever Y is a function of many interval variables [10]. When $Y = f(X_1, X_2, \ldots, X_n)$ is continuous in the n-dimensional convex region, and no extreme point exists in this region (including the boundaries), then the value of the interval function can be obtained by

$$Y=f(X_1, X_2, \ldots, X_n) = \{\min_j [f(c_j)], \max_j [f(c_j)]\};\ j=1, n \qquad (10)$$

where $c_j = (X_{1j}, X_{2j}, \ldots\ldots X_{nj})$ represents the coordinates of the j^{th} vertex in n-dimensional space.

Comparison of Methods by Numerical Example

The amplitude and phase of the complex frequency response function (FRF) are important characteristics of actuator-to-sensor transfer functions for purposes of control-structure interaction. As a means of helping to evaluate the relative merits of the three alternative methods for propagating uncertainty presented in this section, numerical examples are presented for the amplitude of a complex FRF [11].

The equation of motion for a single-degree-of-freedom system subjected to a harmonic disturbing force f is given by,

$$m(d^2x/dt^2) + c(dx/dt) + kx = f(t) \qquad (11)$$

where m is the mass, c the damping coefficient, k the stiffness, x the displacement, and t is time. The amplitude of the complex FRF is given by the familiar formula

$$A(\Omega) = [(k - m\Omega^2)^2 + (c\Omega)^2]^{-1/2} \qquad (12)$$

where Ω is the frequency content of the force f. If it is assumed that m, c and k are all random variables with the probability density functions shown in Figure 1, where m_o, c_o and k_o correspond, respectively, to the nominal or mean value of m, c and k, one can write Equation (12) in the dimensionless form

$$A(m', c', k') = \{k' - m' \Omega'^2)^2 + 4\, \zeta_o^2\, c'^2\, \Omega'^2]\}^{-1/2} \qquad (13)$$

where: $m' = m/m_o$, $c' = c/c_o$, $k' = k/k_o$, $\Omega' = \Omega/\omega_o$

and where: $\omega_o = \sqrt{k_o/m_o}$; $\zeta_o = c_o/(2\sqrt{k_o m_o})$

Despite the simplicity in appearance of Equation (13), an analytic closed-form expression for the derived distribution of $A(\Omega)$ given the distributions of m, c and k is extremely difficult to obtain. Consequently, numerical methods are sought.

To apply the First Order Method, one must first differentiate Equation (13) with respect to m', c' and k'. These derivatives, $\partial A/\partial m'$, $\partial A/\partial c'$, $\partial A/\partial k'$ are quite complicated and have been documented elsewhere [12]. Then one must derive the mean and standard derivation of each of the normalized density functions in Figure 1. The mean in each case is simply unity. For the present example [11], the standard deviations for m', c' and k' (see Figure 1) are

$$\sigma_{m'}^2 = 0.0204, \quad \sigma_{c'}^2 = 0.2458, \quad \sigma_{k'}^2 = 0.0612$$

Finally, if lognormal distributions are assumed for the three uncertain parameters, the distribution functions for various frequency ratios (Ω') shown in Figure 2 are obtained.

These are approximate distributions, which should be good as long as the excitation frequency is not near resonance. In Figure 2, therefore, the plot for $\Omega' = 1$ may not be a good approximation. To verify this assumption, Monte Carlo simulations were run for the same four cases. The results of these simulations based on a sample size of 10,000, i.e., 10,000 evaluations of Equation (13), are superposed on the previously derived lognormal density functions in Figure 2 for comparison. As expected, the first-order approximations are valid for the three off-resonant cases.

If the structural parameters are estimated by triangular fuzzy sets which are similar in shape to the density functions given in Figure 1, the uncertainty in m, c and k (the base of the triangles) are the same in magnitude, but the peak membership at m_0, c_0 and k_0 are normalized to unity to provide for normal membership functions [9]. This can be done because in fuzzy sets, the area under the membership function does not have to be unity as is required for a probability density function. The processing required by Equation (10) is carried out using the Vertex method as previously described.

The derived fuzzy membership functions of FRF amplitude for the four excitation frequencies are shown in Figure 3. The curves in Figure 3 are similar to their counterparts in Figure 2, both in the spread and the frequency at which maximum amplification occurs. Comparison of the absolute density values (ordinates) in Figure 2 with the membership values (ordinates) in Figure 3 is not meaningful because of the theoretical differences between membership functions and density functions [9].

As a final example, all three uncertainty propagation methods are applied to a case where $c' = 0.025$ and $\Omega' = 0.975$. This frequency ratio corresponds to the lower half power point of the frequency response function. The uncertainty in damping is assumed to be negligible for purposes of demonstration. At this frequency, the sensitivity of FRF amplitude to mass and stiffness is greatest, so that the first-order approximation should be at its worst. The results are plotted in Figure 4. Figure 4a shows the comparison between the First Order Method and Monte Carlo Simulation. Figure 4b shows the corresponding membership function. The unusually shaped distribution produced by the Monte Carlo Simulation is evidently the result of the (sightly) rounded peak of the FRF at an amplification of 20, which allows sampled amplitudes to "collect" in this narrow frequency band.

The example in Figure 4 illustrates how the first order method can yield unrealistically high response levels when $A(\Omega)$ is evaluated near resonance. It represents a deliberate attempt to force such a result, and is admittedly a pathological case. In reality, there will be damping uncertainty which will tend to extend the upper tail of the actual distribution, making the first order method a better approximation. However, in general, there is no guarantee that this will happen; the fuzzy set method therefore serves as a limiting check on the first-order method.

When using the vertex method, the number of required FRF calculations is given by:

$$n = 2N_a N_f N_r (2^{N_p}) \tag{14}$$

where N_a = number of alpha cuts
N_f = number of frequencies
N_r = number of FRF's
N_p = number of uncertain parameters

This number, n, can become very large as N_p becomes large. Since the basic uncertain parameters in the present analysis are modal mass and stiffness parameters, N_p depends on the number of modes represented in the generic uncertainty model. For for a system with only 10 modes, N_p can be shown to be equal to 110 and n from Equation (14) is on the order of 10^{33}! However, since the fuzzy set method proves most helpful near resonance, only a few of the uncertain modal parameters should be important in these cases. A method to identify which of the modal parameters are important near resonance would be most desirable.

FUZZY CLASSIFICATION METHOD

An approach involving fuzzy classification [13, 14] is being explored to identify the most significant modal parameters that affect the FRF near resonance in multi-degree-of-freedom systems. In this approach a finite data set, $X = \{x_1, x_2, \ldots x_n\}$ is defined where each data set corresponds to an uncertain modal parameter. Each data set can be characterized by one or more features. The present analysis is looking at two features for each data set: (i) the coefficient of variation of the uncertain parameter which is obtained from the corresponding diagonal term of the covariance matrix (e.g. S_{rr}) of modal mass and stiffness parameters, and (ii) the sensitivity of the desired response quantity to that parameter (e.g. T_{ur}). The fuzzy classification method partitions these n data sets into c classes where c << n. In this case, the classes or groups might be a group of "important" parameters, a group of "unimportant" parameters, a group of "moderately important" parameters, and so forth, where the fuzzy membership value in a class/group could be a measure of its "importance"; i.e. a membership value of 1 would be important and a value of 0 would be unimportant.

As an example, suppose n = 6 and c = 2. The six data sets might correspond to the six independent modal mass and stiffness parameters of a 2-mode covariance matrix [15]. Define $class_1$ as the important parameters and $class_2$ as the unimportant parameters. The fuzzy classification algorithm begins by making arbitrary "crisp" class assignments for each of the data sets, as

	x_1	x_2	x_3	x_4	x_5	x_6
$class_1$:	1	0	0	1	0	0
$class_2$:	0	1	1	0	1	1

If one were interested in the response at some point on a structure near the first resonant frequency, and modes 1 and 2 were well separated, then one would expect only the diagonal modal mass and stiffness terms associated with the first mode (i.e., data sets x_1 and x_4) to be significant.

Details of the fuzzy algorithm have been described elsewhere [13], but the resulting fuzzy partition might take the following form:

	x_1	x_2	x_3	x_4	x_5	x_6
$class_1$:	.91	.08	.13	.95	.11	.07
$class_2$:	.09	.92	.87	.05	.89	.93

Each column of the fuzzy partition matrix above (denoted as the U matrix) defines the membership of a given data set (uncertain parameter) in each of the two classes. The columns must sum to unity regardless of whether U is a fuzzy or crisp partition matrix. In situations where the membership values are not all close to zero or one, additional classes (i.e., $c > 2$) might be assumed and another classification analysis conducted on the data sets.

The fuzzy classification method may be used directly to select a reduced parameter set for subsequent use in the fuzzy vertex method, or it may be used to construct a fuzzy relation which establishes the degree of relationship to which data set x_i and x_j are related. One such fuzzy relation, R, results from the computation,

$$R = (U*U^T)/n \tag{15}$$

where U is a fuzzy partition matrix segregating n data sets into c classes, and the operation * is algebraic [16]. This relation, R, gives a measure of the relative membership of the data clusters to individual classes. This relation has some special properties: it is always symmetric, the sum of its entries is unity, and the measure of "misclassification" is computed by subtracting the trace of the matrix R from unity. The diagonal elements of R give a measure of the total allocation of membership within a class and the off-diagonal elements yield a measure of the membership allocation between pairs of classes.

CONCLUSIONS

Damping is understood to be a major source of uncertainty in structural analysis. Of particular concern is the fact that damping has heretofore been unpredictable in complex structures; it must be determined experimentally for a prototype structure in an environment similar to that in which response must be predicted. In the case of large space structures, it will be impossible to measure damping directly because of physical limitations on ground testing, and because of the differences (atmospheric, thermal and gravitational) between the earth and space environments. Methods to accurately account for damping uncertainty will significantly improve plant models and afford opportunities for more accurate controllers of motion.

A comparison was made of three alternative methods for propagating uncertainty: the First Order Method, the Numerical Simulation Method and the Fuzzy Set Method. A single-degree-of-freedom mass-spring-dashpot system was selected for this purpose. Triangular probability density functions were defined for the mass, stiffness and damping parameters (m, k and c). Frequency response function (FRF) amplitude for displacement response to a force input was computed for the nominal damping ratio of 2.5%.

It was found that the Fuzzy Set Method bounds the range of possible responses and it provides a valuable limiting check on the First Order Method. The Fuzzy Set Method is a relatively inexpensive alternative to numerical simulation for propagating parameter uncertainty in complex models, whereas numerical simulation becomes prohibitively expensive.

The fuzzy classification method reveals clusters in the data in a multi-dimensional feature space. This automated procedure does not produce labels, vis a viz "important" or "unimportant". These labels have been assigned in a very arbitrary sense considering our limited understanding of damping. The utility in these methods for this problem would be in the area of experimental planning and in numerical code predictive accuracy. The notions of clustering of data in some feature space gives a clue of not only what kind of measurements to take, but maybe where to take them and whether redundant measurements are warranted. These methods eventually can act as a confirmation of behavior once specific sets of conditions are known to lead to some well understood response pattern.

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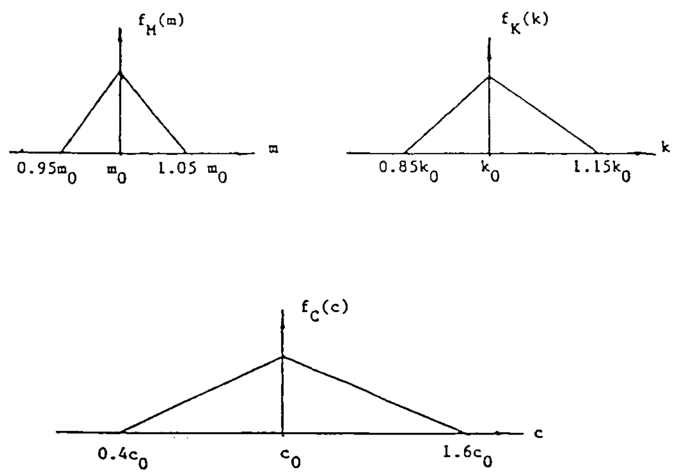


Figure 1. Triangular Forms for Uncertainties in m, k, and c (from [11])

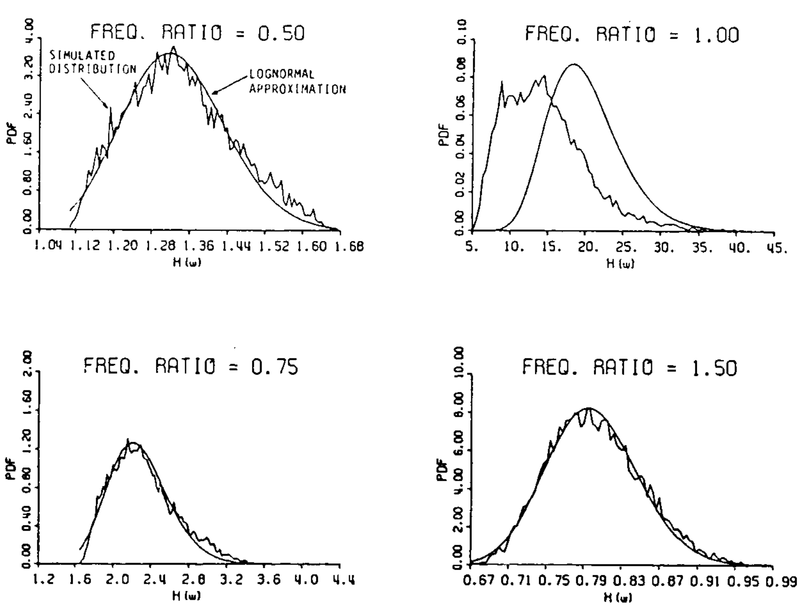


Figure 2. Comparison of First Order Method (lognormal) with Simulation Method (from [11])

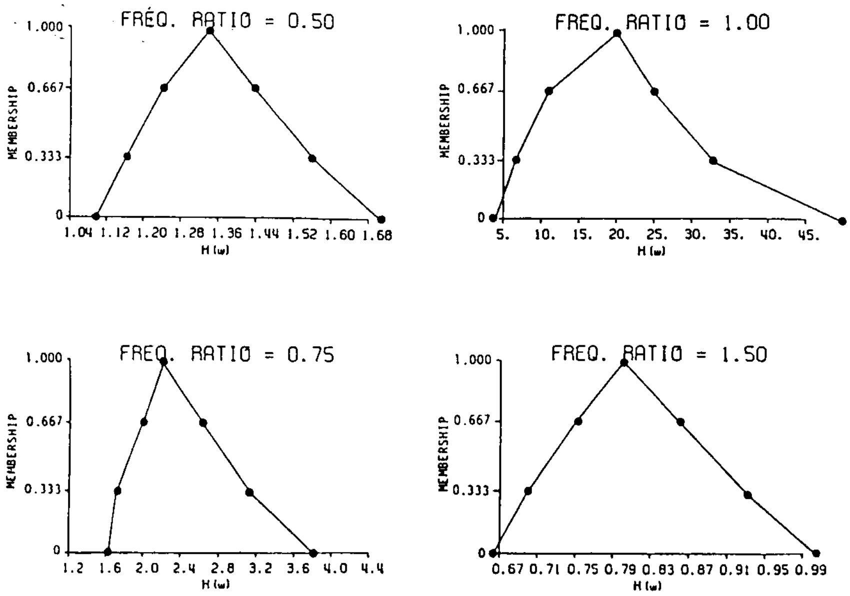
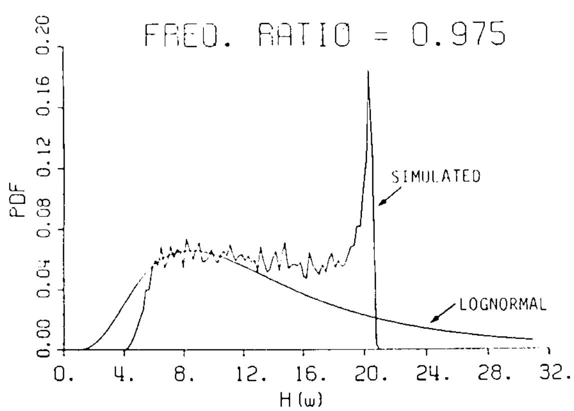
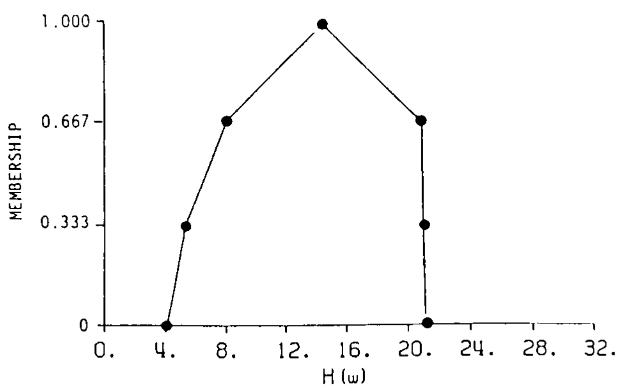


Figure 3. Membership Functions for FRF Amplitude (from [11])



(a) Probability Density Functions.



(b) Membership Function.

Figure 4. Bounding Quality of Fuzzy Sets near Resonance